Doppelte partielle Integration

1a) $\int \underbrace{2x^2}_{u} \underbrace{e^x}_{v'} \, dx = \left[\underbrace{2x^2}_{u} \underbrace{e^x}_{v}\right] - \int \underbrace{4x}_{u'} \underbrace{e^x}_{v} \, dx$ (*)

$= \left[2x^2 e^x\right] - \left[\left[4xe^x\right] - \left[4e^x\right]\right] = e^x \cdot (2x^2 - 4x + 4)$

(*) Nebenrechnung:

$\int \underbrace{4x}_{u} \underbrace{e^x}_{v'} \, dx = \left[\underbrace{4x}_{u} \underbrace{e^x}_{v}\right] - \int \underbrace{4}_{u'} \underbrace{e^x}_{v} \, dx$

$= \boxed{\left[4xe^x\right] - \left[4e^x\right]} = e^x \cdot (4x - 4)$

1b) $\int \underbrace{-2x^2}_{u} \underbrace{e^{2x}}_{v'} \, dx = \left[\underbrace{-2x^2}_{u} \underbrace{e^{2x} \cdot \tfrac{1}{2}}_{v}\right] - \int \underbrace{-4x}_{u'} \cdot \underbrace{\tfrac{1}{2} e^{2x}}_{v} \, dx$ (*)(*)

$= \left[-1x^2 e^{2x}\right] - \left[\left[-2x \cdot \tfrac{1}{2} e^{2x}\right] - \left[-\tfrac{1}{2} e^{2x}\right]\right]$

$= \left[-1x^2 e^{2x}\right] - \left[-1xe^{2x}\right] \cancel{+} - \left[-\tfrac{1}{2} e^{2x}\right]$

$= e^{2x} \cdot \left(-1x^2 + 1x - \tfrac{1}{2}\right)$

(*)(*) Nebenrechnung

$\int \underbrace{-4x \cdot \tfrac{1}{2}}_{=-2x} e^{2x} dx = \int \underbrace{-2x}_{u} \underbrace{e^{2x}}_{v'} \, dx = \left[-2x \cdot \tfrac{1}{2} e^{2x}\right] - \int -2 \cdot \tfrac{1}{2} e^{2x} \, dx$

$= \left[-2x \cdot \tfrac{1}{2} e^{2x}\right] - \left[-\tfrac{1}{2} e^{2x}\right]$

2a) $$\int_0^2 \underbrace{x^2}_{u}\underbrace{e^{2x}}_{v'}\,dx = \left[x^2\cdot\frac{1}{2}e^{2x}\right]_0^2 - \int_0^2 2x\cdot\frac{1}{2}e^{2x}\,dx \quad (*)$$

$$= \left[x^2\cdot\frac{1}{2}\cdot e^{2x}\right]_0^2 - \left(\left[\frac{1}{2}xe^{2x}\right]_0^2 - \left[\frac{1}{4}e^{2x}\right]_0^2\right)$$

$$= \left[\left(\frac{2^2}{2}\cdot e^4\right) - 0\right] - \left(\left[\left(\frac{1}{2}\cdot 2e^4\right) - 0\right] - \left[\frac{1}{4}\cdot e^4 - \frac{1}{4}e^0\right]\right)$$

$$= 2e^4 - \left(e^4 - \frac{1}{4}e^4 + \frac{1}{4}\right) = \frac{5}{4}e^4 - \frac{1}{4} \approx 67{,}998$$

(*) Nebenrechnung: $$\int_0^2 \underbrace{x}_{u}\underbrace{e^{2x}}_{v'}\,dx = \left[\underbrace{x}_{u}\cdot\underbrace{\frac{1}{2}e^{2x}}_{v}\right]_0^2 - \int_0^2 \underbrace{1}_{u'}\cdot\underbrace{\frac{1}{2}e^{2x}}_{v}\,dx$$

$$= \left[\frac{1}{2}xe^{2x}\right]_0^2 - \left[\frac{1}{4}e^{2x}\right]_0^2$$

2b) $$\int_0^1 \underbrace{-2x^2}_{u}\underbrace{e^{-x}}_{v'}\,dx = \left[-2x^2\cdot(-1)e^{-x}\right]_0^1 - \int_0^1 -4x\cdot(-1)e^{-x}\,dx \quad (*)$$

$$= \left[+2x^2e^{-x}\right]_0^1 - \left(\left[-4xe^{-x}\right]_0^1 - \left[4e^{-x}\right]_0^1\right)$$

$$= \left(2e^{-1} - 0\right) - \left(\left(-4\cdot e^{-1}\right) - 0 - \left(4e^{-1} - 4e^{-0}\right)\right)$$

$$= 2e^{-1} + 4e^{-1} + 4e^{-1} - 4 = 10e^{-1} - 4 \approx -0{,}321$$

(*) Nebenrechnung: $$\int_0^1 4xe^{-x}\,dx = \left[4x\cdot(-1)e^{-x}\right]_0^1 - \int_0^1 4\cdot(-1)e^{-x}\,dx$$

$$= \left[-4xe^{-x}\right]_0^1 - \left[4e^{-x}\right]_0^1$$

2c) $$\int_1^2 \underbrace{(x^2+x)}_{u}\cdot\underbrace{e^{3x}}_{v'}\,dx = \left[\underbrace{(x^2+x)}_{u}\cdot\underbrace{\frac{1}{3}e^{3x}}_{v}\right]_1^2 - \int_1^2 (2x+1)\cdot\frac{1}{3}e^{3x}\,dx$$

$$= \left[\frac{1}{3}(x^2+x)\cdot e^{3x}\right]_1^2 - \left(\left[(2x+1)\cdot\frac{1}{9}e^{3x}\right]_1^2 - \int_1^2 2\cdot\frac{1}{9}e^{3x}\,dx\right)$$

$$= \left[\frac{1}{3}(x^2+x)e^{3x}\right]_1^2 - \left(\left[\frac{1}{9}\cdot(2x+1)\cdot e^{3x}\right]_1^2 - \left[\frac{2}{27}e^{3x}\right]_1^2\right)$$

$$= \left(\frac{1}{3}\cdot(2^2+2)\cdot e^6\right) - \left(\frac{1}{3}\cdot(1^2+1)e^3\right) - \left[\frac{1}{9}\cdot(2\cdot 2+1)e^6 - \frac{1}{9}\cdot(2\cdot 1+1)e^3\right) - \left(\frac{2}{27}e^6 - \frac{2}{27}e^3\right)\right]$$

$$= 2e^6 - \frac{2}{3}e^3 - \left[\frac{5}{9}e^6 - \frac{1}{3}e^3 - \frac{2}{27}e^6 + \frac{2}{27}e^3\right]$$

$$= \frac{41}{27}e^6 - \frac{11}{27}e^3 \approx 604{,}431$$