

WS 12, MLK
17.11.21

Produktregel

$$f(x) = u(x) \cdot v(x) \Rightarrow f'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$$

Bsp: $f(x) = x \cdot e^x$

$$u(x) = 1 \cdot x \\ u'(x) = 1$$

$$v(x) = e^x \\ v'(x) = e^x$$

$$f'(x) = \underbrace{1}_{u'} \cdot \underbrace{e^x}_v + \underbrace{1 \cdot x}_u \cdot \underbrace{e^x}_{v'} = \underline{(1 + 1 \cdot x) \cdot e^x}$$

2. Ableitung: jetzt : $u(x) = 1 + 1 \cdot x$ $v(x) = e^x$
 $u'(x) = 1$ $v'(x) = e^x$

$$f''(x) = \underbrace{1}_{u'} \cdot \underbrace{e^x}_v + \underbrace{(1 + 1 \cdot x)}_u \cdot \underbrace{e^x}_{v'} = \underline{(1 + (1 + x)) \cdot e^x} = \underline{(2 + x) \cdot e^x}$$

$$\text{Bsp: } f(x) = (x^2 + 5) \cdot e^{3x}$$

$$u(x) = x^2 + 5$$

$$u'(x) = 2x$$

$$v(x) = e^{3x}$$

$$v'(x) = 3e^{3x}$$

$$f'(x) = \underbrace{2x}_{u'} \cdot \underbrace{e^{3x}}_v + \underbrace{(x^2 + 5)}_u \cdot \underbrace{3e^{3x}}_{v'} = (2x + (x^2 + 5) \cdot 3) \cdot e^{3x} = \underline{\underline{(3x^2 + 2x + 15) \cdot e^{3x}}}$$

2. Ableitung jetzt: $u(x) = 3x^2 + 2x + 15$
 $u'(x) = 6x + 2$

$$v(x) = e^{3x}$$
$$v'(x) = 3e^{3x}$$

$$f''(x) = \underbrace{(6x + 2)}_u \cdot \underbrace{e^{3x}}_v + \underbrace{(3x^2 + 2x + 15)}_u \cdot \underbrace{3e^{3x}}_{v'} = ((6x + 2) + (3x^2 + 2x + 15) \cdot 3) \cdot e^{3x}$$
$$= ((6x + 2) + 9x^2 + 6x + 45) \cdot e^{3x}$$
$$= \underline{\underline{(9x^2 + 12x + 47) \cdot e^{3x}}}$$

Übungen: 1) $f(x) = 4x \cdot e^{2x}$

2) $f(x) = (3x - 6) \cdot e^{5x}$

3) $f(x) = 3x^2 \cdot e^{4x}$

4) $f(x) = 5 \cdot e^{2x}$

5) $f(x) = 3x \cdot e^x$

$$\begin{aligned}
 3) \quad f(x) &= 3x^2 \cdot e^{4x} \\
 f'(x) &= 6x \cdot e^{4x} + (3x^2) 4e^{4x} \\
 &= e^{4x} (6x + (3x^2) 4) \\
 &= \underline{\underline{e^{4x} (6x + 12x^2)}}
 \end{aligned}$$

$$\begin{aligned}
 U(x) &= 3x^2 \\
 U'(x) &= 6x
 \end{aligned}$$

$$\begin{aligned}
 V(x) &= e^{4x} \\
 V'(x) &= 4e^{4x}
 \end{aligned}$$

2. Abl

$$\begin{aligned}
 f''(x) &= (24x + 6)e^{4x} + (6x + 12x^2) 4e^{4x} \\
 &= e^{4x} ((24x + 6) + (6x + 12x^2) 4) \\
 &= e^{4x} ((24x + 6) + (24x + 48x^2)) \\
 &= \underline{\underline{e^{4x} (48x^2 + 48x + 6)}}
 \end{aligned}$$

$$\begin{aligned}
 U(x) &= 6x + 12x^2 \\
 U'(x) &= 24x + 6
 \end{aligned}$$

$$\begin{aligned}
 V(x) &= e^{4x} \\
 V'(x) &= 4e^{4x}
 \end{aligned}$$

$$\textcircled{4} \quad f(x) = 5 \cdot e^{2x}$$

$$u(x) = 5$$

$$u'(x) = 0$$

$$v(x) = e^{2x}$$

$$v'(x) = 2e^{2x}$$

$$f'(x) = (0 \cdot e^{2x}) + (5 \cdot 2e^{2x})$$

$$= 10 \cdot e^{2x}$$

$$u(x) = 10$$

$$u'(x) = 0$$

$$v(x) = e^{2x}$$

$$v'(x) = 2e^{2x}$$

$$f''(x) = (0 \cdot e^{2x}) + (10 \cdot 2e^{2x})$$

$$= 20 \cdot e^{2x}$$